39. Find the shortest distance from the point (2, 1, −1) to the plane $x + y - z = 1$.

$$(d)^2 = \left(\sqrt{(x-2)^2+(y-1)^2+(x+y)^2}\right)^2$$

$$D = (x-2)^2+(y-1)^2+(x+y)^2$$

$$D_x = 2(x-2)+2(x+y) = 0 \rightarrow y = 2-2x$$
$$2x-4+2x+2y=0$$

$$D_y = 2(y-1)+2(x+y)=0 \rightarrow x = 1-2y$$
$$2y-2+2x+2y=0$$

$$x = 1-2(2-2x)$$
$$-3x = -3$$
$$x = 1$$
$$y = 0$$

$$D_{xx} = 4$$
$$D_{yy} = 4$$
$$D_{xy} = 2$$

$$D_{xx}D_{yy} - (D_{xy})^2 > 0$$
$$4 \cdot 4 - 2^2 > 0 \checkmark$$

$D_{xx} > 0 \checkmark$ when $x=1$, $y=0$, min

$$d = \sqrt{(1-2)^2+(0-1)^2+(1+0)^2}$$
$$= \sqrt{3}$$

41. Find the points on the cone $z^2 = x^2 + y^2$ that are closest to the point $(4, 2, 0)$.

$$d = \sqrt{(x-4)^2 + (y-2)^2 + (x^2+y^2)}$$

$$D = (x-4)^2 + (y-2)^2 + (x^2+y^2)$$

$$D_x = 2(x-4) + 2x = 0 \quad 4x - 8 = 0 \quad x = 2$$

$$D_y = 2(y-2) + 2y \quad 4y - 4 = 0 \quad y = 1$$

$$D_{xx} = 4$$

$$D_{xy} = 0$$

$$D_{yy} = 4$$

$$D_{xx} D_{yy} - (D_{xy})^2$$

$$16 > 0$$

$D_{xx} > 0$ min at ~~(1, 2)~~ $(2, 1)$

43. Find three positive numbers whose sum is 100 and whose product is a maximum.

$x+y+z=100$ $\quad$ $z=100-x-y$

$xyz \rightarrow xy(100-x-y) \rightarrow 100xy-x^2y-xy^2$

$f_x=100y-2xy-y^2$ $\quad$ $-2xy=y^2-100y$ $\quad$ $y^2-100y=x^2-100x$

$f_{xx}=-2y \rightarrow y$ must be positive $\quad$ $x=y$

$f_y=100x-x^2-2xy$ $\quad$ $-2xy=x^2-100x$

$100y-2y^2-y^2=0$

$100y-3y^2=0$

$y(100-3y)=0$

$y=0$ $\quad$ $y=\frac{100}{3}$

$x=\frac{100}{3}$